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Research Paper

Projectile penetration into ballistic gelatin

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ABSTRACT

Ballistic gelatin is frequently used as a model for soft biological tissues that experience projectile impact. In this paper we investigate the response of a number of gelatin materials to the penetration of spherical steel projectiles (7 to 11 mm diameter) with a range of lower impacting velocities (<120 m/s). The results of sphere penetration depth versus projectile velocity are found to be linear for all systems above a certain threshold velocity required for initiating penetration. The data for a specific material impacted with different diameter spheres were able to be condensed to a single curve when the penetration depth was normalised by the projectile diameter. When the results are compared with a number of predictive relationships available in the literature, it is found that over the range of projectiles and compositions used, the results fit a simple relationship that takes into account the projectile diameter, the threshold velocity for penetration into the gelatin and a value of the shear modulus of the gelatin estimated from the threshold velocity for penetration. The normalised depth is found to fit the elastic Froude number when this is modified to allow for a threshold impact velocity. The normalised penetration data are found to best fit this modified elastic Froude number with a slope of 1/2 instead of 1/3 as suggested by Akers and Belmonte (2006). Possible explanations for this difference are discussed.

1. Introduction

The investigation of the material behaviour of soft tissues at high impact rates is critical to achieve an understanding of ballistic and explosive related particulate wounding, as well as the forensic pathological interpretation of such wounds. Gelatin has for many years been used as a mechanical simulant for soft biological tissue especially to model impact and penetration of projectiles. In military circles, ballistic gelatin has been widely used for the evaluation of high speed projectiles (bullets) penetrating soft biological tissues (Kieser et al., 2013; Thali et al., 2002; Koene and Papy, 2011). It is also used to simulate the in vivo response for traumatic brain injury (e.g. Alley et al., 2011). Gelatin has also been associated with the development of medical imaging such as ultrasonic scanning and shear wave elasticity imaging (Amador et al., 2011). In these latter studies the strains developed by the ultrasonic waves were relatively small whereas those

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developed under impact conditions are much larger, especially where penetration occurred.

A major advantage associated with the use of gelatin is that it is relatively cheap and easy to manufacture. In addition, it is transparent and by changing its concentration, a range of stiffness values can be reproducibly manufactured. In this manner it is possible to simulate a wide range of biological soft tissues ranging from brain to muscle. However, there are limited useful relationships available to link the depth of penetration to the projectile dimensions, mass and velocity on the one hand, and the mechanical properties of the gelatin on the other. A classic text on impact by Goldsmith (1960, 2001) devotes an entire chapter to elastic contact parameter interactions dealing primarily with Hertzian contact in the elastic range, or the onset of plastic deformation. In another chapter penetration and perforation of thin metallic specimens is considered. More recently, there has been a report that specifically addresses the impact of spherical projectiles into ballistic gelatin (Sigletes, 2008). The resultant expression developed to interpret such data uses the so-called “Poncelet” relationship. Koene and Papy (2011) consider the penetration of large polymeric projectiles, including spheres, impacting gelatin as a basis for understanding the response of non-lethal weapon injuries. These authors relate the penetration of the projectiles to the kinetic energy, momentum and kinetic energy density in an attempt to determine whether the impact will have lethal consequences.

An alternate approach for the evaluation of the penetration is the analysis of impact of fluidic or granular materials by spheres, such as reported by Akers and Belmonte (2006) and Ciamarra et al. (2004). A further extension of this approach is to consider the solid as an elastic-viscous material or a Bingham solid as considered by de Bruyn and Walsh (2004). In the area of water and jet impact of solids, gelatin is often added to water to provide stable stationary drops and cylinders that are impacted at high velocity so that the resultant stress waves and pressures developed can be assessed (Lesser and Field, 1983; Field et al., 2012).

Recently Kwon and Subhash (2010) and Subhash et al. (2012) have investigated the high strain rate (to 3000/s) compressive and shear responses of gelatin using a modified split Hopkinson bar approach. These authors reported a strong strain rate dependence on the measured stress–strain response of gelatin with a 1000 fold increase in the yield response at strain rates above 2000/s. They also found that gelatin using a shear analysis approach may be considered a strongly shear rate sensitive (hardening) material.

In the present paper, initially the relevant background analysis for the contact and penetration of spheres into various media are presented. This is followed by the results generated by a simple pneumatic system using a range of spherical projectiles from 7 to 11 mm diameter penetrating four different gelatin compositions.

2. Basic relationships

2.1. Elastic contact

For an elastic solid the contact of a sphere of radius R with a semi-infinite half space was developed by Hertz (1882). The mean contact pressure is given by (Goldsmith, 1960)

$$P_m = \frac{F_m}{\pi a_m^2} = 0.1677\left[\frac{v_o^2}{(\delta_1+\delta_2)(\delta_1+\delta_2\,)^4}\frac{m_1}{R_1^3}\right]^{1/5}. \tag{1}$$

where F_m is the maximum force, a_m the maximum contact radius, v_o the impact velocity, m_1 the projectile mass $((m_1 = (\rho 4\pi R_1^3/3))$, ρ is the projectile density, δ_1 and δ_2 are the elastic details $(\delta = ((1-\mu^2)/\pi E))$ with μ the Poisson's ratio and E the Young's modulus.

If we assume that prior to penetration the response of the gelatine is entirely elastic with an E modulus much less than the projectile (steel) and Poisson's ratio of 0.5, with the Shear modulus $G=E/3$, then the critical contact pressure from Eq. (1) may be written

$$P_m = 0.29\vartheta_o^{2/5}\rho^{1/5}G^{4/5} \tag{2}$$

The interesting implication of this expression is that the contact pressure is independent of the impacting ball diameter. If this contact pressure is associated with the yield stress of the material then the value of P_m at the onset of permanent deformation is equivalent to the hardness of the material.

An alternative approach, assuming that the gelatin is a liquid like material, is to consider the maximum pressure generated to be determined by the so called “water hammer pressure”. This approach is common for the impact of liquid droplets and cylinders hitting a rigid object and has formed the basis for an extensive analysis of liquid drop impact and erosion studies (Lesser and Field, 1983, Field et al., 2012). The water hammer pressure is given by:

$$P_{mw} = k\rho C v \tag{3}$$

where k is an acoustic impedance parameter, ρ is the fluid density, C the shock wave velocity in the fluid and v is the impact velocity. The acoustic impedance of the two bodies on impact is discussed in Graf (1975) which, for the case of gelatin and steel, results in k having a value of approximately 0.1. The maximum pressure generated by the droplet impact is also determined by the time taken for the release waves from the edge of the impact to cause flow and jetting at the edge of the droplet. This is typically generated over a contact radius given by $R=rv/C$) and has a duration related to the radius of contact and the shock wave velocity ($t_m \sim rv/2C^2$) (Field et al., 2012). In addition, the shock wave velocity is related to the impact pressure. Typical values of C for water (and small concentrations of gelatin) are 1.50 km/s. Once flow of the drop or fluid commences the pressure reduces to the Bernoulli stagnation pressure $P=rv^2/2$.

2.2. Impact with penetration

Following the seminal work of Poncelet (1829) and most recently that of Goldsmith (1960), Allen et al. (1956) and others, a general empirical expression relating the force developed as a function of impact velocity of a projectile of mass m is given by:

$$F = -m\frac{dv}{dt} = B_1v^2 + B_2v + B_3v \tag{4}$$

where B_1, B_2 and B_3 are constants to be identified. At low velocity the penetration depth of a projectile (assuming $B_1=B_2=0$) into a solid is given by $h=(m/2B_3)v^2$.

Whereas at higher velocities $B_1 \neq 0$ and the depth of penetration is given by

$$h = \frac{m}{2B_1} \ln\left(\frac{B_1}{B_3}v^2 + 1\right) \tag{5}$$

From studies of blunt nose projectiles striking sand at low velocities the term $B_1 = (1/2)C_D\rho A v^2$ where ρ is the target density, A the frontal (cross-sectional) area and C_D is the drag coefficient. For a Newtonian fluid $C_D=2$, whereas for elastic contact between grains of sand $C_D=4$.

An alternate approach is to consider the resistance of the projectile as the resistance of a Newtonian liquid in which case, from the Stokes relationship the viscous drag is given by $3\pi Dv\eta$, where η is the viscosity of the medium

An extension to the above approach is to consider the media being impacted as a Bingham solid that behaves elastically up to its yield stress and thereafter as a Newtonian viscous material. This approach was initially developed by Ansley and Smith (1967) and more recently taken up by de Bruyn and Walsh (2004). The latter authors investigated the penetration of steel balls into spherical glass media of different diameters, whereas the former studied genuinely Bingham materials and the penetration response of different diameter spheres. de Bruyn and Walsh (2004) considered the response of their glass particles as initially elastic and at a certain force the "yield" stress of the assembled particles was exceeded and the sphere was able to penetrate the particles. Under these conditions the authors assumed the force on the moving spherical projectile was given by,

$$F = -F_y - \alpha\eta D v \tag{6}$$

where F_y is related to the "yield" stress and the additional term is the viscous drag with α a constant equal to 3π. To determine the depth of penetration of the projectile these authors integrated the above expression and upon determining the time the velocity equals zero they find the depth of penetration is given by

$$h = \frac{mv_o}{\alpha\eta D} - \frac{F_y m}{(\alpha\eta D)^2} \ln\left\{1 + \frac{(\alpha\eta D)v_o}{F_y}\right\} \tag{7}$$

The first part of the above expression is due to the viscous drag while the second part is attributable to the target material's yield stress. As pointed out by de Bruyn and Walsh (2004), if the initial velocity is large enough, the influence of the logarithmic term is small and the depth of penetration versus projectile velocity is almost a straight line.

We now wish to consider a more recent analysis of spherical projectiles penetrating into visco-elastic fluids by Akers and Belmonte (2006). These authors explored the penetration depth achieved in terms of classic fluid mechanics terms including the Reynolds R_e, Deborah D_e and Weber W_b numbers, which were defined as follows:

$$R_e = \frac{v d \rho_f}{\eta}, \quad D_e = \frac{v\lambda}{d}, \quad W_b = \frac{d\rho_f v^2}{\gamma} \tag{8}$$

where d is a spatial dimension, ρ_f the density of the fluid, λ a relaxation time and γ the surface tension. They found, however, that the depth of penetration for a wide range of spheres penetrating a specific visco-elastic fluid was best matched with the so called elastic Freude number F_e which may be defined as,

$$F_e = \frac{(\rho_s - \rho_f)v^2}{G} \tag{9}$$

where ρ_s is the impacting solid density. They found that the normalised depth of penetration ($h/2R$) scaled with $F_e^{1/3}$ for a wide range of spheres of different density and impacting velocities. The implication of the above is that it is the elastic response of a visco-elastic material that plays a primary role in determining the depth of penetration of impacting objects.

The different expressions outlined above will be applied to the experiments about to be described.

3. Experimental details

3.1. Method

The experimental apparatus consisted of a pneumatic argon based ballistic device, Fig. 1. The clear PVC barrels could be exchanged to accommodate the different projectiles (7, 9 and 11 mm diameter). The velocity of the impacting spheres was measured with a shooting chronograph (F1 Shooting Chrony, Chrony Inc. USA) which was positioned under the tubing so that the pre-collision velocity of the projectiles could be measured.

3.2. Materials

The gelatin blocks used in the initial experiments were made from 250B ordnance gel (Gelita NZ), using a 7.5% by weight mixture kept at 4 °C. Details of the preparation of the gelatin followed the approaches used by various authors and are discussed in detail in Kieser et al. (2013), Jussila (2004) and Fackler and Malinowski (1988). Blocks of dimensions 7.5 × 16 × 16 cm were refrigerated for 24 h at 4 °C prior to testing. The blocks were removed from their moulds and testing completed on each block within 15 min, so as to ensure that temperature fluctuation did not affect results. An additional study was made of the penetration into 5, 10 and 15% concentrations of gelatin from a different source.

The second set of experiments also used Gelita 250 Bloom (bovine) gelatin source and were fabricated with concentrations containing 5, 10 and 15% gelatine. These were prepared at

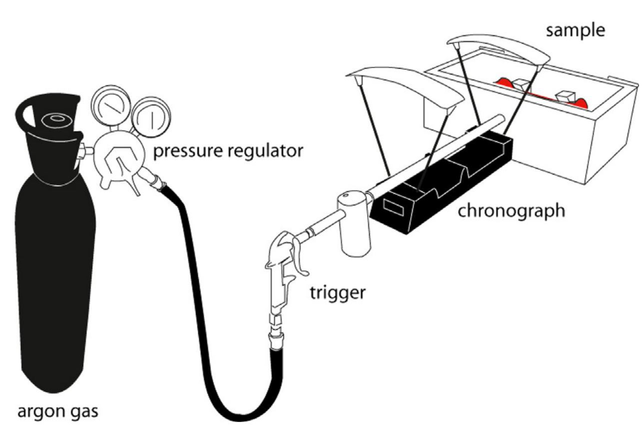

Fig. 1 – A schematic of the experimental setup. Diagram reproduced with permission of illustrator, Mr R. Riddell.

<58 °C and then left to gel at room temperature for 24 h, before moving them to the refrigerator (4 °C) to condition for another 24 h before commencing the penetration tests Shah (2013).

The projectiles used were hardened spherical steel balls of 7, 9 and 11 mm diameter with mass of 1.400, 2.974 and 5.596 g respectively.

4. Results

The 3 different sized projectiles were fired at a range of velocities into the different gelatin materials, with both the initial velocity just prior to impact and depth of penetration recorded. Multiple shots could be fired into each block, provided there was no overlap of permanent or temporary cavities (Kieser et al., 2013). Penetration of the projectiles was easily measured by inserting a thin metal rod into the permanent cavity until the rod came into contact with the embedded projectile (Fig. 2). The diameter of the projectile was then added to the measured length to give the total penetration of the leading edge of the ball.

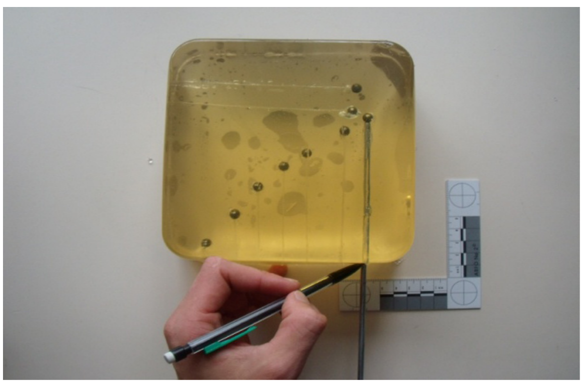

Fig. 2 – An example of the ballistic gelatin blocks used for testing and the method of depth of penetration measurement.

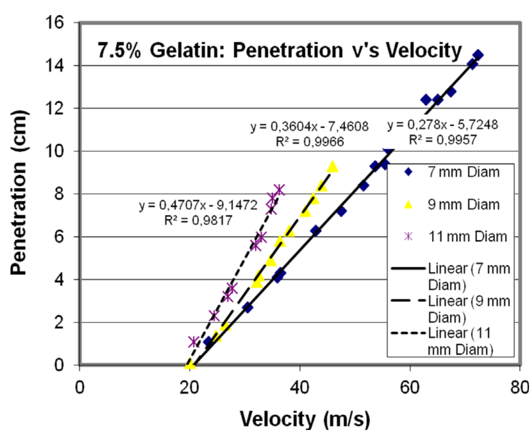

Fig. 3 – Data and linear best fit lines for penetration vs. impact velocity for each of the projectiles used for the 7.5% gelatin material. The velocity at which each line intersects zero penetration on the y-axis indicates the value of the threshold velocity (V_{tsh}) for each projectile, below which penetration does not occur.

Typical observations of the depth of penetration into the 7.5% gelatin blocks as a function of impact velocity for the 3 balls are shown in Fig. 3. The data generated appeared to fit a linear relationship between depths of penetration and impacting velocity with the largest diameter ball penetrating deepest for the same velocity. In all instances there was a threshold velocity below which penetration did not occur. The relationship between penetration and initial velocity could be well approximated by a linear relationship of the form

$$h = Av + B \tag{10}$$

where A and B are constants that differ for each projectile, h is the penetration and v is the projectile velocity.

The results of the penetration depth relationships (A and B), the goodness of fit (R^2) of the linear fit and the threshold velocity (V_{th}) for impact into the gelatin are given in Table 1. It was found that the threshold velocity for each projectile, below which penetration did not occur for the 7.5% gelatin, was almost constant at 20 ± 0.9 m/s.

In the case of the different gelatin concentrations impacted with the 9 mm diameter sphere similar behaviour to that observed in Fig. 3 are shown in Fig. 4. As seen in this

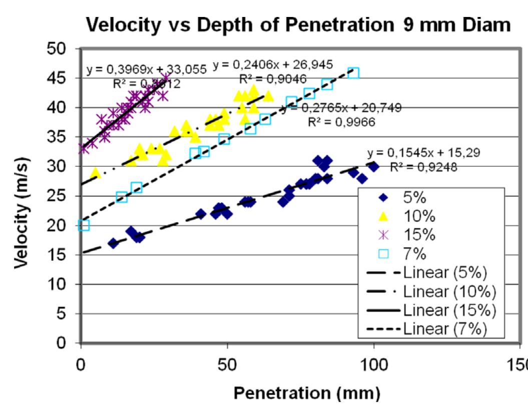

Fig. 4 – Plots of the velocity of impact versus the penetration depth of a 9 mm diameter projectile into blocks of the 4 different gelatin concentrations. In all instances the data is well fitted by straight lines.

Table 1 – Displays the A and B coefficients for the penetration plotted against velocity equation of the form $p = Av + B$ for each projectile, along with the R^2 values and threshold velocities v_{th}. The contact force and pressure at the threshold velocity are also estimated.

Projectile (mm)	Mass (g)	A	B	R^2	V_{tsh} (m/s)	Impact force at v_{th} (N)	Contact pressure at v_{th} (MPa)
7	1.4001	0.278	−5.725	0.996	20.59	42.9	1.11
9	2.9735	0.360	−7.461	0.997	20.70	70.7	1.11
11	5.5960	0.471	−9.147	0.982	19.43	109	1.15

figure the intercept or threshold velocity for penetration was dependent on the volume % of gelatin and the slopes of the lines were also different. The position of the 7.5% gelatin results appears to match the threshold velocity but the slope appears to be somewhat greater than expected as it is higher than the 10% gelatin specimen.

The threshold velocities for projectile penetration were 15.3, 26.9 and 33.1 m/s for the 5, 10 and 15% gelatin concentrations respectively while that of the 7.5% composition was 20.7 m/s.

5. Discussion

The results shown in Figs. 3 and 4 suggest a near linear relationship between penetration depths with increasing projectile velocity above a specific threshold, for all systems investigated. The threshold velocity was almost identical for the 7.5% gelatin with the different diameter projectiles at ~20 m/s. At higher impact velocities the depth of penetration was greater for the bigger diameter ball. The threshold velocity for penetration increased with gelatin concentration for the 9 mm diameter projectile as shown in Fig. 4. The above outcomes are very similar to the extensive observations by Jussila (2004) for penetration of a 4.5 mm diameter spherical particle. The presence of the residual cavity created by the penetrating ball indicates that the materials have a definite yield stress as otherwise the cavity would have collapsed entirely as observed by Akers and Belmonte (2006) with their penetration studies on visco-elastic fluidic materials. We did, however also observe that a residual cavity was present for all the gelatine materials, with a smaller diameter than the sphere diameter. This suggested that substantial contraction after passage of the projectile had occurred although this appeared to be stable with time for several hours after projectile impact.

The ability to use the expression in Eq. (1) for determination of the critical contact pressure requires knowledge of the Shear modulus G for the different gelatin materials which unfortunately was not measured during the course of these experiments. An approximate estimate of the contact pressure developed for a sphere impacting gelatine may be had from the threshold velocity conditions as observed in the results. The force generated may be estimated from the relationship $F=ma$, where m is the mass of the sphere and a is the deceleration. If we make the assumption that at the threshold velocity there is no rebound and the impacting sphere comes to rest, then from simple Newton's laws of motion relationships we have $v^2=u^2+2as$, where v is the final velocity, u the initial velocity a the acceleration and s the distance over which the acceleration occurs. For elastic contact of a sphere on a flat surface, the depth of penetration of the contact diameter is half the total displacement. If we consider at the threshold the contact diameter is the same as the impacting sphere diameter then the distance travelled to come to rest is also the sphere diameter. The resultant estimated impact forces at the threshold velocity for the different gelatins and impacting spheres are given in Table 1. On this basis the contact pressure may also be estimated from the impact force divided by the contact area (projected cross-sectional area of the spheres) and these are listed in Table 1. For the 7.5% gelatin material with a threshold velocity of 20.7 m/s the value of the contact pressure is 1.13 ± 0.02 MPa. From this value of impact pressure an estimate of the shear modulus G may be had from Eq. (2), resulting in values listed in Table 2.

The values of the effective shear modulus listed for gelatin materials in Table 2 are much higher than the quasi-static values reported by Amador et al. (2011). This may arise because the effective strain rates in the current study at the threshold velocity for penetration are much higher than those investigated by Amador et al. (2011). A simple estimate of the effective strain rate for the impact conditions may again be had from an extension of the approach used above. At the threshold velocity we assume that the sphere has penetrated to its diameter. As pointed out by Tabor (1951) the strain associated with spherical contact with a flat may be approximated by $\varepsilon=a/R$, where a is the contact radius and R is the radius of the sphere. At the impact threshold velocity, the strain ε is ~1. An estimate of the time taken to develop this strain may be had again from Newton's laws of motion namely $v=u+at$, and with $u=20$ m/s, a from above ~20 to 30×10^3 m/s^2 results in a strain rate of greater than 10^3/s. A previous study of the properties of gelatin materials at comparable strain rates (2000–3000/s) was conducted by Kwon and Subhash (2010). The present Shear modulus results are in keeping with the observations of Kwon and Subhash (2010) in that they are almost 1000 times greater than the quasi-static measurements or those reported by Amador et al. (2011). The latter authors show that the quasi-static Shear modulus over the range of 5 to 15% gelatin, ranged from 1.6 to 5.4 kPa and is almost linearly proportional to

Table 2 – Comparison of the threshold velocity for penetration of the 9 mm diameter ball along with the estimated shear modulus of the gelatins and calculated contact pressure (using Eq. (1)) at the threshold velocity and the approach outlined in Section 5.

Gelatin composition (%)	Threshold velocity, v_{th} (m/s)	Shear modulus G (MPa)	Contact pressure at v_{th} (MPa)
5*	15.3	2.17	0.61
7.5**	20.7	3.95	1.13
10*	26.9	6.67	1.88
15*	33.1	10.1	2.84

* Gelatin tested at RT (20 °C).
** Gelatin tested at 4 °C.

gelatin content. The values of the threshold contact pressure for the spheres in contact with gelatin based upon the water hammer pressure expression (Eq. (3)) are typically 2.0 to 4.0 MPa which is again much higher than the quasi-static values of the shear modulus but close to the estimates generated from the analysis above and listed in Table 2. These high values are associated with the near incompressibility of the liquid (gelatin) until release waves or flow is initiated beneath the contact.

The results for the penetration depth versus impact velocity for the 7.5% gelatin impacted with the 3 diameter projectiles (Fig. 3) can be collapsed onto a single line when the penetration depth divided by sphere diameter are plotted versus impact velocity (h/D vs v) as shown in Fig. 5. This type of normalisation has previously been proposed by de Bruyn and Walsh (2004) as well as Akers and Belmonte (2006).

Turning our attention now to the results for the different concentrations of gelatin impacted with the 9 mm diameter ball (Fig. 4) there is a clear trend for the impact velocity threshold increasing with gelatin content. However the slopes of the data, while consistent for the gelatin from a single source, show differences between the sources. That is the slope of the 7.5% gelatin data is greater than the 10%. This may have arisen because the 7.5% results were tested while the gelatin was at relative low temperatures (4 °C) whereas the other compositions were tested at room temperature. In addition there were slight differences in the preparation procedure of the gelatin. The data for all the gelatin materials and all the impacting projectiles are shown in Fig. 6 with the normalised penetration plotted versus the impact velocity minus the threshold velocity for penetration. Not surprising the results of the linear fits pass through the origin. Also shown on this plot are the slopes of the linear fits.

Figs. 5 and 6 both show linear relationships between normalised penetration and reduced velocity. This is not in agreement with the results of Akers and Belmonte (2006), who showed that when they plotted their normalised penetration depths versus the elastic Froude number (Eq. (9)) for the different spheres they used on log–log axes, a linear slope of 1/3 was found. When the results generated in this study were plotted simply as the normalised depth of penetration versus the elastic Froude number, the slopes for each

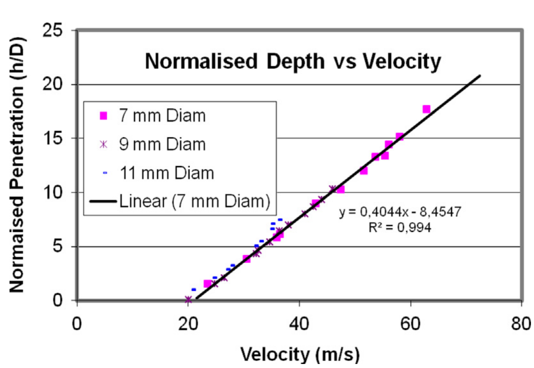

Fig. 5 – Plot of the normalised penetration (depth of penetration divided by sphere diameter) versus impact velocity for the 3 different diameter projectiles impacting the 7.5% gelatin. The data is well fitted by a single linear relationship.

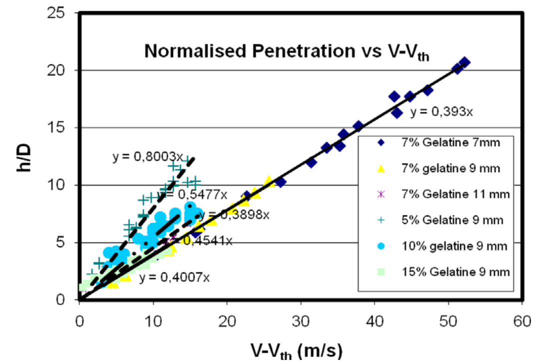

Fig. 6 – Plot of normalised depth of penetration versus the impact velocity minus the threshold velocity for penetration for the 4 different concentrations of gelatin investigated. In all instances the data is well fitted by a linear relationship (equation adjacent to the data).

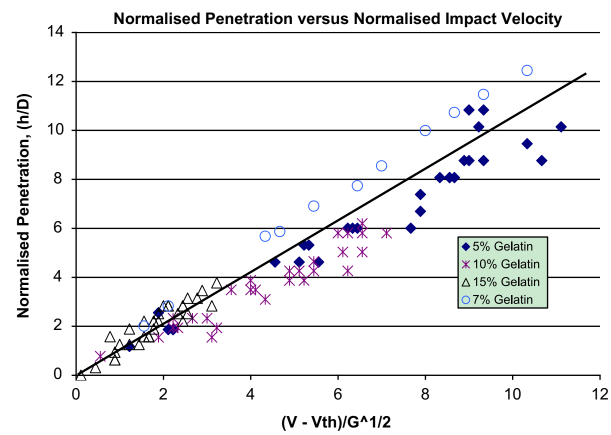

Fig. 7 – Plots of the normalised depth of penetration for all projectiles versus the reduced velocity (Impact velocity minus the threshold velocity for penetration) divided by the square root of the shear modulus. The data superimpose reasonably well for the 4 different concentrations of gelatine and the 3 impacting spheres. The values of the shear modulus are listed in Table 2.

projectile and each gelatine were different and non linear, even when plotted on log–log axes. The results in Figs. 5 and 6 show linear slopes when the normalised penetration was plotted against (V–V_{th}) implying that a better relationship with the modified elastic Froude number is a slope of 1/2 as the projectiles all had the same density and the difference in density of the gelatin over the gelatin concentrations considered is very small.

As mentioned earlier, while the elastic properties of the gelatins tested were not determined independently, an estimate for G may be obtained from the threshold velocities for penetration of the gelatine as mentioned above. However, based on Eq. (9) the slopes of the linear fits in Fig. 6 should scale with $G^{-1/2}$. Using the values estimated from the threshold velocity for penetration as listed in Table 2, the velocity axis results in Fig. 6 can be normalised by the calculated values for $G^{-1/2}$ as shown in Fig. 7. The observations in Fig. 7

show a reasonable correlation although there are slight differences between the different gelatin materials. These slight differences may have arisen because of skin effects present with the different gelatins.

Comparing the current observations with those of Akers and Belmonte (2006) it is apparent in Fig. 4 of their paper that there is an upward trend of normalised penetration depth (h/R) with increasing Froude number F_e. These authors investigated a much lower shear modulus visco-elastic material (only 22 Pa yield stress) and had lower impact velocities than investigated here plus there was no suggestion of a threshold velocity for penetration. Akers and Belmonte (2006) complimented their paper with images of the penetrating spheres into the visco-elastic fluid they investigate. At low impact velocities (<1 m/s) they found that the spheres generated only minor trailing cavities which collapsed in less than 1 diameter behind the sphere. At higher impact velocities (>2 m/s), however, they found well developed cavities that extended for substantial distances behind the penetrating spheres before eventually collapsing. In the current study none of the cavities produced by the projectiles collapsed although they were significantly narrower than the diameter of the impacting projectile. Akers and Belmonte (2006) argued that if the impacting sphere stopped before the cavity closed, then based upon an energy balance (the kinetic energy of the projectile equals the energy to empty the cavity column of material), a scaling law between the normalised penetration depth and the elastic Froude number, namely $(h/D)\sim F_r^{1/2}$ was appropriate. This is precisely the relationship observed in the current investigation when the velocity of the projectile is corrected for the observed threshold to penetrate the gelatin.

A critical factor in the above is that the materials have been assumed to behave in either a Bingham like or an elastic–plastic response. Akers and Belmonte (2006) also noted that the spheres in their study rebounded within the visco-elastic medium and the final depth of penetration was less than the maximum depth observed from high speed photographic results. This may have happened in the current study and will be the basis for further investigation.

Our results clearly identify the role of a threshold velocity for penetration into the gelatin. This does not appear to be a function of the radius of the sphere provided the composition is not changed. This strongly supports the simplified Hertz analysis as given in Eq. (2), although the value derived for the contact pressure and shear modulus values derived from the impact threshold are much higher than the quasi-static estimates of the shear modulus of similar compositions as measured by Amador et al. (2011) [5]. The values of G are similar to those recently reported by Kwon and Subhash (2010) from high strain rate impact tests at comparable strain rates to those estimated in this study. The water hammer pressure estimates for the contact pressure are similar to those estimated from the threshold velocity.

Of the various expressions canvassed for interpretation of the penetration into the gelatin the Bingham visco-elastic model appears to be relevant as it accounts for a threshold velocity to exceed a yield stress. If for instance, the material was a simple viscous body, then according to de Bruyn and Walsh (2004) h should scale as $v_0/\eta D$. The expression given in Eq. (7) appears to be of the form required to rationalise the observed results if the term containing F_y is considered to relate to the threshold velocity. However, it is not straightforward to use this expression and again there would not be the h/D normalisation of the results that is observed. The Poncelet expression, Eq. (4), which both Sigletes (2008) and de Bruyn and Walsh (2004) utilise is an empirical relationship and has a number of terms that need to be determined. The approach by Akers and Belmonte (2006) uses the Froude expression, a methodology with a long tradition in fluid mechanics literature. However, these authors, based upon simple energetic arguments and collapsing of the cavity behind the penetrating sphere, rationalise a relationship in terms of their so called elastic Froude number Eq. (9). The current results, however, were found to fit a different relationship than Akers and Belmonte (2006) found experimentally for a Bingham material with a very low yield point and no detectable threshold velocity for penetration. On the other hand the results were in good agreement with a simple energetic argument that Akers and Belmonte (2006) develop for a material that does not have a collapsing cavity, provided the velocity of impact is corrected by subtracting the threshold velocity for penetration.

6. Conclusions

Gelatin appears to be a highly repeatable medium for impact penetration studies at relatively low velocity. There is clear evidence for a threshold velocity of penetration that is dependent upon the concentration of gelatin and for spheres of the same material appears to be independent of the radius, as expected from Hertz's elastic analysis of impact. The penetration depth normalised by the diameter of the projectile is a convenient method to condense results generated with different spheres. Simple application of Newton's Laws of motion enabled an estimate of the Shear modulus of the different gelatins from the threshold velocity for penetration. These values were much higher than quasi-static estimates. The so called elastic Froude number appears to be a simple means of collapsing the results onto almost a universal curve for a number of gelatin compositions.

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